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TECHNICAL NOTES

Combined heat and mass transfer in turbulent natural convection between vertical parallel plates

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INTRODUCTION

ENERGY transport in natural convection flows driven by the combined buoyancy forces of heat and mass transfer resulting from the simultaneous presence of differences in temperature and variations in concentration are important in natural environments and engineering applications. Prominent examples include double-diffusive convection in ocean flows, the simultaneous diffusion of metabolic heat and perspiration in controlling our body temperature especially on hot summer days, and the cooling of a high-temperature surface by coating it with phase-change material and the process of evaporative cooling for waste heat disposal.

The natural convection heat transfer in a vertical open channel induced by the buoyancy force of thermal diffusion alone has been examined for laminar flows [1–3] and turbulent flows [4–6].

The effects of the mass diffusion on laminar natural convection flows have been widely studied for flows over plates with different inclinations [7, 8] and for flows over vertical cylinders [9]. Recently, laminar natural convection heat and mass transfer in a vertical channel was investigated by Lee *et al.* [10] and Chang *et al.* [11].

The reviews mentioned above for the analysis of heat and mass transfer in natural convection flows, however, restricted their considerations to laminar natural convection flows. At high Grashof numbers the natural convection flow undergoes transition from laminar to turbulent flow. In this connection it is worthwhile to study the combined heat and mass transfer in turbulent natural convection flow.

In this note an extension of the related works [10, 11] is made to investigate the heat and mass transfer in turbulent natural convection flow of an air–water vapour mixture in a finite vertical plate channel. Particular attention is paid to examining the extent of the heat transfer enhancement through mass diffusion.

ANALYSIS

The geometry to be examined is a vertical parallel-plate channel with plate height l and channel width b. The channel wall is wetted by a thin water film. The loss of evaporated water in the film is compensated by the injection of additional water through the porous channel wall. With good control of the water injection and the appropriate choice of the porosity of the porous channel wall, the film on the wall can be maintained extremely thin so that it can be regarded as stationary and at the same uniform temperature as the channel wall T_w. The moist air in ambient with temperature T_0 is drawn into the channel by the combined buoyancy forces due to the nonuniformities in temperature and in concentration of water vapour between the wetted wall and the ambient. By introducing the Boussinesq approximation, the time-averaging turbulent natural convection flow of moist air in a vertical channel resulting from the combined buoyancy effects of thermal and mass diffusion can be described by the basic equation as:

continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0; \tag{1}$$

axial-momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\mathrm{d}p_m}{\mathrm{d}x}+\frac{\partial}{\partial y}\left[(v+v_t)\frac{\partial u}{\partial y}\right] +g\beta(T-T_0)+g\beta^*(w-w_0); \tag{2}$$

energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left[\left(\frac{v}{Pr}+\frac{v_t}{Pr_t}\right)\frac{\partial T}{\partial y}\right] +\frac{(c_{pv}-c_{pa})}{c_p}\left(\frac{v}{Sc}+\frac{v_t}{Sc_t}\right)\frac{\partial T}{\partial y}\frac{\partial w}{\partial y}; \tag{3}$$

equation of species diffusion for water vapour

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\frac{\partial}{\partial y}\left[\left(\frac{v}{Sc}+\frac{v_t}{Sc_t}\right)\frac{\partial w}{\partial y}\right] \tag{4}$$

where the usual boundary layer approximation has been invoked. In addition, the thermophysical properties of the mixture are taken to be constant and are computed at the film conditions. This special way of evaluating the properties is found to be appropriate for the study of the combined heat and mass transfer problem [12]. The complete details on the evaluation of properties of air, water vapour and their mixture are given by Fujii *et al.* [13].

The last two terms on the right-hand side of equation (2) are the buoyancy forces due to thermal and mass diffusion. The volumetric coefficients of expansion β and β^* are

$$\beta = 1/T_0 \quad \text{and} \quad \beta^* = (M_a/M_v-1). \tag{5}$$

The governing equations are subjected to the following boundary conditions:

$$\begin{aligned} &x=0\,(\text{inlet}): u=u_0, T=T_0, w=w_0, p_m=-\rho u_0^2/2\\ &y=0\,(\text{centreline}): \partial u/\partial y=\partial T/\partial y=\partial w/\partial y=0\\ &y=b/2\,(\text{wall}): u=0, v=v_w(x), T=T_w, w=w_w(x)\\ &x=l(\text{outlet}): p_m=0. \end{aligned} \tag{6}$$

It is worth noting that the dynamic pressure at the inlet is determined by the condition $p_m=-\rho u_0^2/2$ instead of zero, which is to account for the pressure drop in the ambient just outside the inlet [3]. In addition, the transverse interfacial velocity of the mixture, v_w, and mass fraction of water

NOMENCLATURE

b	channel width
c_p	specific heat
c_1, c_2, c_μ	constants appearing in the k–ε turbulence model
D	mass diffusivity
f_1, f_2, f_μ	functions appearing in the k–ε turbulence model
Gr_M	Grashof number (mass transfer), $g(M_a/M_v-1)(w_r-w_0)b^3/v^2$
Gr_{Mx}	mass transfer Grashof number, equation (18)
Gr_T	Grashof number (heat transfer), $g\beta(T_w-T_0)b^3/v^2$
Gr_{Tx}	heat transfer Grashof number, equation (17)
g	gravitational acceleration
h_{fg}	latent heat of vaporization
k	turbulent kinetic energy
l	channel length
M	molecular weight
Nu_l	local Nusselt number (latent heat)
Nu_s	local Nusselt number (sensible heat)
Nu_x	overall local Nusselt number
p	pressure of the mixture in the channel
p_m	motion pressure (or dynamic pressure), $p-p_0$
p_0	ambient pressure
p_w	partial pressure of water vapour at interface
Pr	Prandtl number
Pr_t	turbulent Prandtl number for T
Q	total heat transfer rate
Q_0	total heat transfer rate without liquid water film
q''_w	interfacial energy flux flowing into air stream
Re	Reynolds number based on hydraulic diameter
S	parameter, equation (14)
Sc	Schmidt number
T	temperature
u	velocity
$\overline{u'T'}$	longitudinal turbulent heat flux
$\overline{u'w'}$	longitudinal turbulent mass flux
v	transverse velocity
$\overline{v'T'}$	transverse turbulent heat flux
w	mass fraction of water vapour
w_r	saturated mass fraction of water vapour at T_w and P_0
x	coordinate in the flow direction
y	coordinate in the transverse direction.

Greek symbols

β	thermal expansion coefficient, $1/T_0$
β^*	concentration expansion coefficient
ε	rate of dissipation of turbulent kinetic energy
λ	molecular thermal conductivity
v	molecular kinematic viscosity
v_t	turbulent eddy viscosity
ρ	density
$\sigma_k, \sigma_\varepsilon$	turbulent Prandtl number for k and ε, respectively
ϕ	relative humidity of air at the ambient condition.

Subscripts

a	air
r	reference condition
v	water vapour
w	condition at interface
0	ambient condition.

vapour, w_w, vary in the streamwise direction [11]. They are

$$w_w = p_w M_v/[p_w M_v + (p-p_w)M_a] \tag{7}$$

$$v_w = -\frac{D}{1-w_w}\left.\frac{\partial w}{\partial y}\right|_{\text{wall}}. \tag{8}$$

Energy transport from the channel wall to the moist air in the presence of mass transfer depends on two related factors: the fluid temperature gradient at the wall and the rate of mass transfer. The connection of energy transfer with these two factors is obtained by means of an interfacial energy balance [14]

$$q''_w = \lambda\left.\frac{\partial T}{\partial y}\right|_{\text{wall}} + \frac{\rho D h_{fg}}{1-w_w}\left.\frac{\partial w}{\partial y}\right|_{\text{wall}}. \tag{9}$$

A local Nusselt number is defined as

$$Nu_x = \frac{q''_w(2b)}{\lambda(T_w-T_0)}. \tag{10}$$

This leads to

$$Nu_x = Nu_s + Nu_l \tag{11}$$

where

$$Nu_s = \frac{2b}{T_w-T_0}\left.\frac{\partial w}{\partial y}\right|_{\text{wall}} \tag{12}$$

$$Nu_l = \frac{2bS}{w_r-w_0}\left.\frac{\partial w}{\partial y}\right|_{\text{wall}}. \tag{13}$$

In equation (13), S signifies the relative importance of energy transport through species diffusion to that through thermal diffusion at the interface

$$S = \rho D h_{fg}(w_r-w_0)/[\lambda(T_w-T_0)]. \tag{14}$$

TURBULENCE MODELLING

In the study of the simultaneous heat and mass transfer occurring in natural convection flows, the transition criteria which characterize the beginning and the end of transition, equations (1-1) and (1-2) in ref. [15], for the thermal turbulent natural convection flows should be modified to account for the additional buoyancy effect of mass diffusion. The modified expression for the beginning of transition is taken to be

$$Re^{1/2}(Pr\,Gr_{Tx} + Sc\,Gr_{Mx})^{1/3} = 1.2\times10^5. \tag{15}$$

Similarly, the end of transition is also modified and indicated by an equation of the same form

$$Re^{1/2}(Pr\,Gr_{Tx} + Sc\,Gr_{Mx})^{1/3} = 1.5\times10^5 \tag{16}$$

where $Re = 2u_0b/v$ is the Reynolds number, and Gr_{Tx} and Gr_{Mx} the local Grashof numbers for heat and mass transfer, respectively, defined as

$$Gr_{Tx} = g(T_w-T_0)x^3/(v^2T_0) \tag{17}$$

$$Gr_{Mx} = g(M_a/M_v-1)(w_r-w_0)x^3/v^2. \tag{18}$$

The choice of the criteria, equations (15) and (16), and the intermittency concept used in the computation of the transition from laminar to turbulent flow are described in great detail in ref. [15].

The turbulent viscosity ν_t is computed in accordance with the k–ε turbulence model. Hence the transport equations for the turbulent kinetic energy and turbulent energy dissipation must be included in the study. To procure more reliable results, a low Reynolds number k–ε model [16, 17] is selected to eliminate the usage of wall functions, which are difficult to derive for natural convection flows, in the computations and thus to permit direct integration of the transport equations to the channel wall.

The turbulent kinetic energy equation is given as

$$u\frac{\partial k}{\partial x}+v\frac{\partial k}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right]+\nu_t\left(\frac{\partial u}{\partial y}\right)^2-\varepsilon-2\nu\left(\frac{\partial k^{1/2}}{\partial y}\right)^2+g\beta\overline{u'T'}+g\beta^*\overline{u'w'} \quad (19)$$

and the rate of dissipation of turbulent kinetic energy is determined from

$$u\frac{\partial \varepsilon}{\partial x}+v\frac{\partial \varepsilon}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right]+c_1f_1\frac{\varepsilon}{k}\nu_t\left(\frac{\partial u}{\partial y}\right)^2-c_2f_2\frac{\varepsilon^2}{k}+2\nu\nu_t\left(\frac{\partial^2 u}{\partial y^2}\right)^2+c_1\frac{\varepsilon}{k}(g\beta\overline{u'T'}+g\beta^*\overline{u'w'}) \quad (20)$$

where

$$\overline{u'T'}=\frac{\nu_t}{Pr_t}\frac{\partial T}{\partial y} \quad (21a)$$

$$\overline{u'w'}=\frac{\nu_t}{Sc_t}\frac{\partial w}{\partial y} \quad (21b)$$

$$\nu_t=c_\mu f_u(k^2/\varepsilon). \quad (21c)$$

The appropriate constants and turbulent Prandtl numbers, Pr_t, and constants σ_k and σ_ε as well as the low Reynolds number wall damping functions, f_1, f_2, and f_u, are the same as those of ref. [15].

RESULTS AND DISCUSSION

In this note the effects of a wetted wall on turbulent natural convection heat transfer are examined in great detail. As already mentioned above, a system literature search has not revealed any theoretical and experimental data for heat and mass transfer in turbulent natural convection flows through a vertical channel to which the results of the present study can be directly compared. To check the applicability of the numerical scheme and the turbulence modelling, however, the results for two limiting cases were first obtained. First, the predicted results from this study for the laminar natural convection through a vertical channel because of the simultaneous presence of combined buoyancy forces of thermal and mass diffusion are in good agreement with the results of Chang *et al.* [11]. Another limiting case is the turbulent natural convection through vertical parallel plates without mass transfer. The computed results are in line with the experimental results of Eckert and Diaguila [4]. This lends support to the employment of the turbulence model and the numerical scheme as proposed above to the study of the present problem. Furthermore, special care is taken to establish the number and arrangement of gridlines required to produce essentially grid-independent results. A total of 101×61 gridlines was placed in the x- and y-directions, respectively. Larger gridlines showed that the Nusselt number results changed by less than 2% with a doubling of the gridlines.

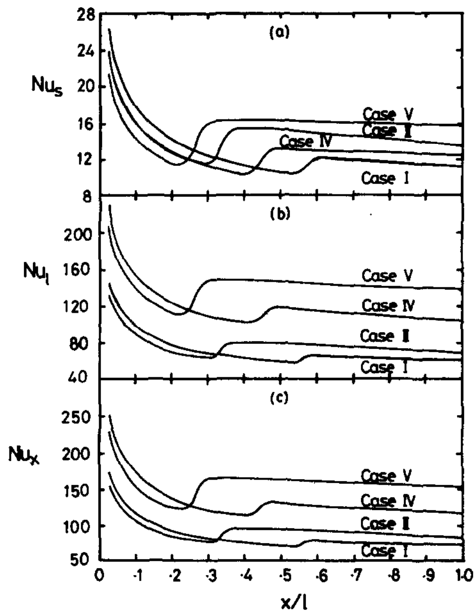

FIG. 1. Local Nusselt numbers for (a) sensible heat, (b) latent heat, (c) total along the channel. Case I, T_w = 40°C, l = 2.5 m, ϕ = 50%; Case II, T_w = 40°C, l = 4 m; Case IV, T_w = 60°C, l = 2.5 m; Case V, T_w = 60°C, l = 4 m.

In this note the calculations are especially performed for the moist air flowing in the channel, a situation widely found in engineering systems. Other mixtures can be analysed in the same way. It should be emphasized that not all the values for the non-dimensional groups, e.g. Pr, Sc, Gr_T, Gr_M, etc., can be arbitrarily assigned. In fact, they are interdependent for a given mixture under certain specific conditions. In the light of practical situations, the following conditions are selected in the computations: unsaturated moist air with a relative humidity of 50% at 20°C and 1 atm is driven into a vertical parallel plate channel with a channel width b = 0.08 m and with different lengths by the buoyancy forces of heat and mass diffusion; the channel walls are assumed to be at the same uniform temperature varying from 40 to 60°C. All the non-dimensional parameters can then be evaluated. Results are obtained for several cases presented in Table 1.

To demonstrate the relative effects of heat transfer through sensible heat and latent heat exchange in the flow, three kinds of Nusselt numbers are presented in Fig. 1. Careful scrutiny

Table 1. Values of major parameters for various cases

Case	T_w	l	ϕ	Pr	Sc	$Gr_T = g\beta(T_w-T_0)b^3/\nu^3$	$Gr_M = g\beta^*(w_r-w_0)b^3/\nu^2$	S
I	40	2.5	50	0.704	0.593	1.36×10^6	4.77×10^5	5.419
II	40	4	50	0.704	0.593	1.36×10^6	4.77×10^5	5.419
III	50	2.5	50	0.703	0.591	1.93×10^6	8.30×10^5	6.507
IV	60	2.5	50	0.701	0.589	2.45×10^6	1.36×10^6	8.282
V	60	4	50	0.701	0.589	2.45×10^6	1.36×10^6	8.282

Units for parameters: T in °C, l in m, ϕ in %.

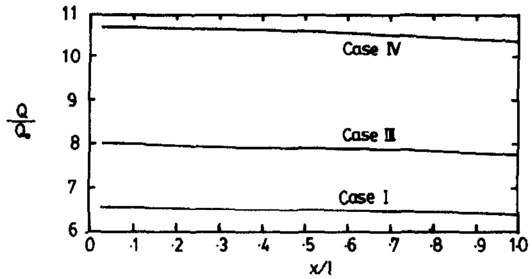

FIG. 2. Effects of system temperatures on the total heat transfer rate. Case I, $T_w = 40°C$, $l = 2.5$ m, $\phi = 50\%$; Case III, $T_w = 50°C$; Case IV, $T_w = 60°C$.

of these plots shows that the flow near the channel entrance is still laminar and the heat transfer coefficient decreases with x due to the entrance effect. But as the air moves downstream, the natural convection flow near the heated walls becomes unstable. Through a series of complicated mechanisms of linear and non-linear interactions, the flows become fully turbulent. In this transitional stage the heat transfer coefficient increases with x. As the flow becomes fully turbulent, the augmentation in heat transfer stops. Instead, heat transfer gradually decreases. Furthermore, it is also found that the region of flow transition is shifted downstream as the channel wall temperature is lowered or the channel is shortened. Regarding Nu_s-curves, in the laminar regime a larger Nu_s is found for the flow with a lower wall temperature T_w. This is a direct consequence of the findings by Chang *et al.* [11]—larger water vapour evaporation associated with higher T_w causes the temperature profiles to become flattened, which then results in less sensible heat transfer. An opposite tendency is observed in the turbulent regime—Nu_s increases with T_w. This is because the higher the wall temperature, the larger the amount of air driven into the channel resulting from the combined buoyancy forces of heat and mass diffusion, that is, the flow has higher Reynolds number for higher T_w for a fixed channel length. Accordingly, the sensible heat transfer increases with *Re*, which confirms the general perception that the turbulent convection heat transfer process is more effective in the flow with a higher Reynolds number. The effect of channel length on Nu_s has the same trend as the wall temperature, shown by comparing the curves for cases I and II and for cases IV and V.

Attention is turned to the Nu_l distributions (Fig. 1(b)). According to the results in Fig. 1(b) the flow with a higher wall temperature shows a higher value for Nu_l. This is brought about by the larger latent heat transport in connection with the larger liquid film evaporation for higher T_w. By comparing the ordinate scales of Figs. 1(a) and (b), it is apparent that the magnitude of Nu_l is much larger than that of Nu_s, which indicates that heat transfer resulting from latent exchange is much more effective. Also clearly seen in this plot is the influences of the flow transition on the variations of Nu_l. Figure 1(c) shows the total Nusselt number distributions (i.e. $Nu_x = Nu_s + Nu_l$).

It is physically important to be aware of the effect of latent heat transfer through mass diffusion. This can be demonstrated by comparing the results of the total heat transfer from the wetted wall to the moist air with the corresponding results for the situation in which the channel wall is dry. The results are given in Fig. 2, here Q_0 represents the total heat transfer rate under the corresponding conditions for different cases except no liquid film on the channel surface. It is clearly seen that Q/Q_0 can be as large as 10 for $T_w = 60°C$. Thus latent heat transport predominates over sensible heat transfer.

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